Power-law cross-correlations in econophysics: New insights

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Abstract

Power-law cross-correlations in econophysics are most frequently studied purely empirically. In our contribution, we provide a theoretical treatment of several "stylized facts" or "laws" of such correlations and we stress that these need to be treated and analyzed very carefully before any strong conclusions are made.

Keyword: power-law cross-correlations, econophysics

Ever since the introduction of the detrended cross-correlation analysis (DCCA) for studying long-range (power-law) cross-correlations in 2008 [1, 2], the method has quickly found its home in applications to financial time series and the discipline of econophysics has found a new area of research. Various other methods for analyzing the long-term relationship between a pair of series have been developed and applied since then [3–5]. The literature has majorly grown in the empirical applications and analysis of various types of financial and economic series (the methods have been used in other disciplines as well).

There are three major findings in the empirical literature. Firstly, the bivariate Hurst exponent H_{xy}, which measures cross-persistence between two analyzed series, is frequently found to be above the level of no long-term memory, i.e. $H_{xy} > 0.5$ and power-law cross-correlations are thus identified. Such feature is then interpreted as a sign of a significant temporal relationship between the analyzed series. Secondly, the bivariate Hurst exponent is also frequently close to an average of Hurst exponents H_x and H_y of the separate series, i.e. $H_{xy} \approx \frac{1}{2}(H_x + H_y)$. And thirdly, the bivariate Hurst exponent is sometimes estimated above the average of the separate Hurst exponents, i.e. $H_{xy} > \frac{1}{2}(H_x + H_y)$, and such finding is usually interpreted as a sign of some stronger interaction between analyzed series.

In our contribution, we tackle these three findings from a theoretical perspective. Firstly, we show that cross-persistence very easily arises from long-term memory in the separate processes. In fact, only one series from the analyzed pair needs to be power-law auto-correlated and the cross-persistence emerges practically automatically. Secondly, we argue that for standard classes of processes such as ARFIMA and AR processes (or their combinations), the equality $H_{xy} = \frac{1}{2}(H_x + H_y)$ holds almost always. And thirdly, we demonstrate that $H_{xy} \leq \frac{1}{2}(H_x + H_y)$ for standard processes so that $H_{xy} > \frac{1}{2}(H_x + H_y)$ is not feasible and it can be only attributed to an inefficient estimation of the bivariate Hurst exponent.

We thus stress that power-law cross-correlations in econophysics need to be treated and analyzed very carefully before any strong conclusions are made.

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