A Hidden Mechanism Governing Synchronization Limit and Tsallis Entropy Maximization

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Abstract

Nonlinear oscillators exhibit synchronization (injection-locking) to external periodic forcings, which underlies the mutual synchronization in networks of nonlinear oscillators. Despite its history of synchronization and the practical importance of injection-locking to date, there are many important open problems of an ideal injection-locking for a given oscillator. In this work, I elucidate a hidden mechanism governing the synchronization limit under weak forcings, and the Tsallis entropy maximization, which is related to a widely known inequality; Hölder's inequality. This mechanism enables us to understand how and why the ideal injection-locking is realized; a general theory of synchronization limit is constructed where the maximization of the entrainment range or the stability of entrainment for general forcings including pulse trains, and a fundamental limit of general m:n entrainment, are clarified systematically. In addition, this mechanism provides a clear-cut understanding to the Tsallis entropy maximization. Furthermore, practical design principles for efficient injection-locking (and hence, better mutual synchronization) are extracted from the general theory of entrainment limit, whose utility is verified in the Hodgkin-Huxley neuron model as an example.

Keyword: synchronization, entrainment, injection-locking, fundamental limit, Tsallis entropy, Arnold tongue, optimization, Hölder's inequality