Financial Brownian Particle in the Layered Order-Book Fluid from a point of view of statistical physics

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Abstract

With massive data analysis, we have established a fundamental analogy between the motion of a colloidal particle embedded in a fluid and the price dynamics of a financial market in the order book. Even though the market evolves by decision-making process of traders, the Langevin equation, a stochastic differential equation that describes the colloidal motion in a molecular fluid, is also applied to the fluctuation of its behaviors (i.e., traders ' reactions to place their limit orders against price changes).

Keyword: Brownian motion, financial market, order book information

A qualitatively novel type of model for financial price fluctuations is introduced in order to account for the numerous stylized facts observed in order book information, the aggregation of placed orders by traders, of financial market. For that, we propose the picture that the observed financial motion is analogous to a genuine colloidal Brownian particle embedded in a fluid of smaller particles. The picture is empirically based on the results of analyzing the order book data of the Electronic Broking Services for currency pairs, (we use the Dollar-YEN, Euro-Dollar, and Euro-Yen, market) provided by a market managing company ICAP, the foreign exchange market of which is continuously open 24 hours per day except over weekends with large liquidity.

The order book information is evolved by injection of three types of orders; limit orders, market orders and cancelation. A limit buy (resp. sell) order is introduced by a trader by specifying the buying (resp. selling) price and its quantity. A market buy (resp. sell) order is injected with quantity but without the price information, and it directly hits against a sell (resp. buy) order at the best ask (resp. bid) causing a deal. A cancelation simply deletes an order, which can be done only by the trader who placed the order. The number of orders at a certain price changed by those orders has an non-random characteristic, which is revealed by the cross correlation funciton between price changes and number changes of limit orders.

Regarding the characteristic, intuitively, when the price goes up the number of newly injected buy (resp. sell) orders near the best price increase (resp. decrease) as a force pushing (resp. pulling) the market price into up side, and also there is a tendency that the number of newly injected sell orders far from the best price increase as a force to make the direction stop, which is interpreted to be caused by the traders' expectation of getting larger return assuming continuation of the same price movement. It is mentioned as one of conclusions that the relation between price movements and the collective behavior of traders in limit orders movements is interpreted as the Langevin equation describing the movements of Brownian partilce in a molecular fluid.

The proposed quantitative correspondence provides a novel perspective for the analysis of financial markets. Our approach demonstrates the importance of physical intuition associated with financial insights in analyzing the big data of financial markets.

References

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