Fractional Dynamics of Human Memory

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Abstract

Appealing to the available psychological and physiological data we propose a model of long-term human memory assuming its multi-trace structure. The pivot point is a mathematical description of the creation of new memory traces caused by learning a certain fragment of information pattern and affected by the fragments of this pattern already retained by the current moment of time. The final equation governing the learning and forgetting processes is constructed in the form of the differential equation with the Caputo type fractional time derivative. Several characteristic situations of the learning (continuous and discontinuous) and forgetting processes are studied numerically.

Keyword: Human Memory, Forgetting, Learning, Practice, Fractional Differential Equations

Model and Basic Assumptions

Using the advances made in psychological and physiological investigations of human memory [1, 2] we accept the following basic postulates.

1. The declarative (long-term) memory is organized in chunks, certain cognitive units related to some information objects.

2. Each chunk individually is characterized by its strength F which determines also the information retention ability and on its own decays with time t according to the power-law

$$F(t) \propto \frac{1}{t^d},$$

   where the exponent d is a certain constant estimated as $d \sim 0.1 - 0.2$.

3. Human memory is of the multiple-trace arrangement, so each attempt of learning and memorizing some information fragment produces a separate trace m in human memory and the strength F of corresponding memory unit (chunk) is the sum of their individual activation levels

$$F(t) = \sum_{m \in \text{Chunk}} F_m(t).$$

4. Due to the multi-trace consolidation, when a new memory trace overlapping with the original ones is created only its non-overlapping fragment is stored in the memory.

Based on these postulates we developed the following model for the single chunk dynamics. Some learning process of a certain pattern $\mathbb{P}$ is initiated at time $t = 0$ and before it no information about $\mathbb{P}$ was available, i.e., for $t < 0$ the value $F(t) = 0$. It is demonstrated that this process is governed by the fractional differential equation of the Caputo type

$$\tau^{(1-d)} \cdot {}^{C}D^{(1-d)} F = (\epsilon + F)^g (1 - F)^\gamma W(t) \,. \quad (1)$$

Hear the "microscopic" time scale τ, the coefficient ϵ, and the exponents g, γ are system parameters, the function $W(t)$ quantifies the human attention during the learning process.

Results and Discussion

The characteristic features of the system dynamics were studied numerically, Figure 1 presents some of the obtained results.

Plot I shows the forgetting dynamics. As should be expected, the asymptotics of $F(t)$ is of the power law and looks like a straight line on the log-log scale plot. Naturally, in a certain neighborhood $\mathcal{Q}_L$ of the time moment $t = T_L$ this asymptotics does not hold. However, for small values of the exponent d (for $d = 0.2$ in Plot I) this neighborhood is narrow and becomes actually invisible in approximating experimental data even with weak scattering.

Plot II exhibits the learning dynamics. As seen, the function $F(t)$ strictly is not of the power law. However, if it is reconstructed from some set of scattered experimental points as the best approximation within a certain class of functions, a power law fit (linear ansatz in the log-log scale) may be accepted as a relevant model. It allows us to introduce an effective exponent d_L of the approximation

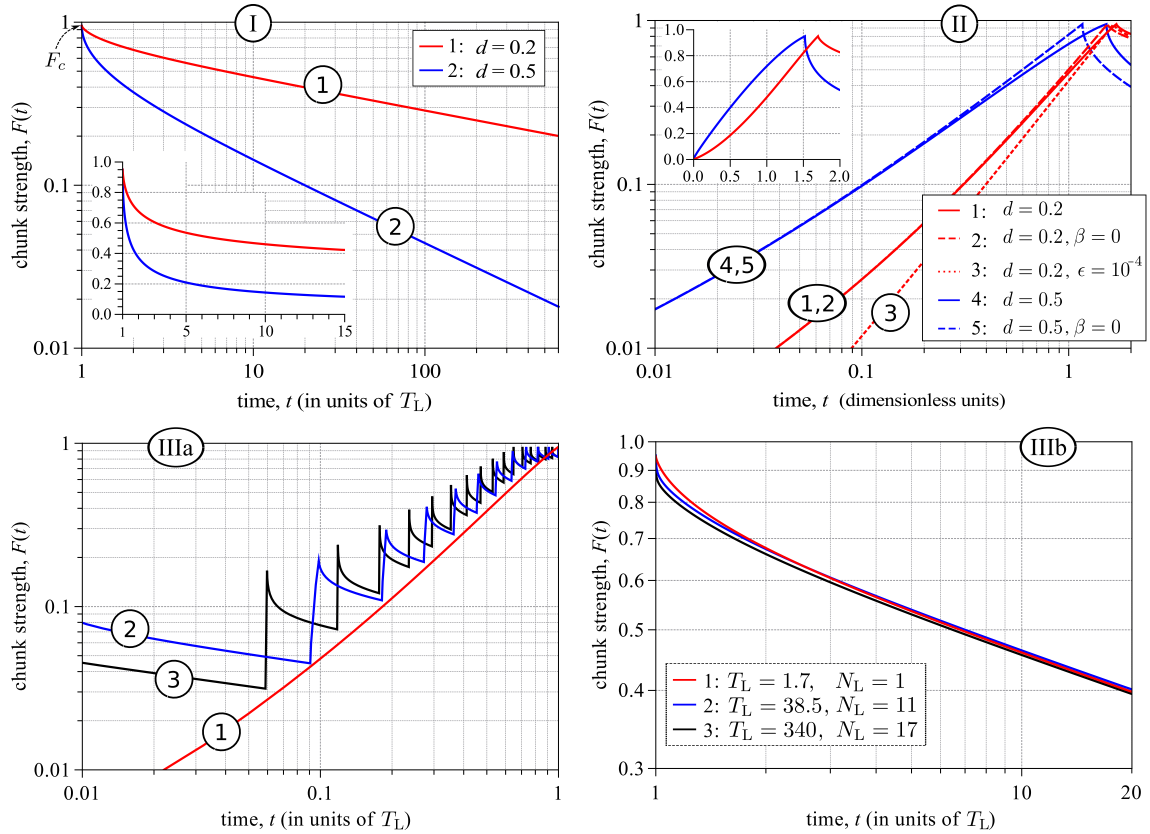

Figure1: Some results of numerical simulation. First, it is the dynamics of forgetting (I) and learning (II) under the "basic" conditions when a subject learns an unknown information continuously until he gets the local (temporal) success in time T_L. Second, it is the spacing effects in discontinuous learning (IIIa) and the following forgetting (IIIb). In this case the subject learns an unknown information via a sequence of trials of a fixed duration until he gets the final success at a certain moment of time T_L. Naturally, the longer the time spacing between two successive trials, the larger the number N_L of trials and the longer the total time interval T_L are necessary for this. In simulations the following parameters $\epsilon = 0.01$, $\alpha = 0.5$, $\beta = 0.1$, $F_c = 0.95$, $dt = 0.001$, and $T_\mathrm{L}|_{d=0.2} = 1.70$, $T_\mathrm{L}|_{d=0.5} = 1.52$ were used as a common setup, the other individual values are shown in the corresponding plots.

$F(t) \propto t^{d_L}$. Appealing again to Plot II, we draw a conclusion that this effective exponent depends not only on the "forgetting" exponent d but also on the other system parameters. Thereby, in trying to determine the set of quantities required for characterizing human long-term memory, the "forgetting" and "learning" exponents, d and d_L, may be regarded as independent parameters.

Plots IIIa and IIIb illustrate the found results in the case mimicking the discontinuous learning process. In this case, as seen in Plot IIIa and IIIb, the main characteristics of the shown processes become rather similar with respect to the dynamics of learning and forgetting. This result poses a question about optimizing a learning process by dividing it into rather short trials separated by relatively long time intervals.

Also we note appealing to the obtained results that the multiple trace concept of memory architecture requires an individual mathematical formalism irreducible to the classical notions created in physics. In particular, even at the "microscopic" level dealing with slides (traces) the system dynamics is not reduced to the motion in a certain phase space but continuous generation of such phase spaces. Their interactions with one another become a key point of the corresponding theory.

References

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