Transfer Entropy: Game Theory and Theoretical Markets

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Abstract

How can we measure the temporal directed influence one subsystem has on another in a complex and strongly interacting setting? It is often important for us to know how information flows through a complex system such as a financial market where we need to distinguish between the historical contribution of an equity's price and that of another equity's historical contribution to the current price. In a different situation we may want to measure the behaviour of an economic agent A playing a two player, two choice game by looking at the influence the other player's behaviour has on A's choice while excluding the influence of A's past. This work will present the results of using Transfer Entropy in these two very different economic situations where we use models of financial markets and behavioural monkey data from micro-economic game theory.

Keyword: Transfer Entropy, Game Theory, Financial Markets

1 Transfer Entropy

Transfer Entropy was developed by Schreiber [1] as a rigorous way of measuring the directed transfer of information from one stochastic process to another after accounting for the history of the primary process (see below) for arbitrary distributions. This is a natural extension of Granger Causality, first introduced by Granger [2] in econometrics, in which Gaussian processes are assumed. This work applies the transfer entropy to two novel processes.

A statistical process generates a temporal sequence of data: $\mathbf{X} = \{\ldots, x_{t-1}, x_t\}$, X_t is a random variable with possible states S_X, $x_t \in S_X$ and $\mathbf{X}_t^k = \{x_{t-k}, \ldots, x_{t-1}\} \in \{S_X\}^{k-1}$ is a random variable called the k-lagged history of X. We can similarly define a second process Y and so we have the joint process $[\mathbf{X}, \mathbf{Y}] = \{\ldots, [x_{t-1}, y_{t-1}], [x_t, y_t]\}$ where $[x_t, y_t] \in S_X \times S_Y$. The marginal probability is $p(X_t)$ and the conditional probability of X_t given its k-lagged history is $p(X_t|\mathbf{X}_t^k)$ and further conditioned upon the second process $\mathbf{Y}_t^k$ is $p(X_t|\mathbf{X}_t^k, \mathbf{Y}_t^k)$. With these definitions we can define the conditional (Shannon) entropies:

$$\begin{aligned} \mathbf{H}(X_t|\mathbf{X}_t^k) &= -\mathbf{E}[\log p(X_t|\mathbf{X}_t^k)], & (1) \\ \mathbf{H}(X_t|\mathbf{X}_t^k, \mathbf{Y}_t^k) &= -\mathbf{E}[\log p(X_t|\mathbf{X}_t^k, \mathbf{Y}_t^k)] & (2) \end{aligned}$$

and the k-lagged *transfer entropy* for $\mathbf{Y}$ to $\mathbf{X}$ is then:

$$\mathbf{T}_{Y \to X} \equiv \mathbf{H}(X_t|\mathbf{X}_t^k) - \mathbf{H}(X_t|\mathbf{X}_t^k, \mathbf{Y}_t^k). \quad (3)$$

$\mathbf{T}_{Y \to X}$ is interpreted as the degree to which X_t is disambiguated by the lagged history of $\mathbf{Y}$ beyond that to which X_t is already disambiguated by its own lagged history. This work presents recent developments in transfer entropy [3], information theory and the 'critical phenomena' of markets [4], some of the recent success in using it as a predictive measure of the phase transition in the 2-D Ising model [5] and some new results using behavioural monkey data from game theory and financial markets. Future prospects for this approach will be discussed through its application to complex adaptive systems in which the information that *flows* through a system acts as a predictor or even mediator of global changes in behaviour and hence may act as a strong candidate as a *predictive* measure of global changes.

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