

Chances for your team to win the Worldcup 2022

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An Elo ratings based Monte Carlo simulation is performed for FIFA Worldcup Qatar 2022 for your beer-drinking discussions on it.

football | Worldcup | Monte Carlo simulation | Elo ratings

The outcome of FIFA Worldcup Qatar 2022 is of considerable interest of many. The aim of this report is to provide a good reference for discussions on it, based on the Elo rating.

Method

We sample the results by the Monte Carlo method. The probabilities of winning, draw, and lose of a team for each match are estimated from the Elo ratings (1) at October 1st., which is after the last international match day before the Worldcup. We simulate 1 billion samples, i.e. roughly equal to the earth's age if we sample every four years.

Result

Table 1 shows the estimated probabilities of the final results for all the teams those have > 1% probability of winning the cup. The chance to win the cup is dominated by Brazil and Argentina, followed by European teams and Uruguay. We can also see the effect of the grouping and the tournament structure in the slight difference between the rank in Elo rating and the rank in estimated probabilities to reach each stage. Estimated probabilities of all the teams for the final positions including the group league rounds are shown in Table 2.

To mimic the effect of the momentum of the teams after good or bad match result, we also perform the simulation with dynamically updating the Elo rating as $\Delta\text{Elo} = 60(R - P_{win})$, where the result R takes 1, 0.5 and 0 for win, draw, and lose, respectively (1). While this model gives larger probabilities for the winning up of low-rated teams which needs successive upsets, the differences are small (Table 3).

Table 1. Estimated probabilities of reaching to each stage in %, in the descending order of winning probability.

Country	Elo R.	Win	Final	R-4	R-8	R-16
BRA	2169	26.8	37.1	55.7	75.2	94.9
ARG	2141	22.3	32.7	53.9	73.1	97.4
NLD	2040	8.80	16.9	34.8	72.0	95.2
ESP	2045	7.75	15.3	28.6	57.9	87.8
BEL	2025	6.77	15.8	27.6	52.3	87.6
FRA	2005	6.53	16.2	33.0	51.1	88.3
PRT	2004	5.06	12.6	23.5	43.7	87.9
DEN	1971	4.02	11.1	25.2	42.4	83.7
DEU	1960	2.64	7.43	16.1	38.7	73.3
ENG	1920	1.93	6.56	17.1	41.2	75.1
URY	1936	1.79	5.47	12.8	28.9	77.5
CHE	1929	1.50	5.12	11.6	26.7	56.7
HRV	1922	1.37	4.34	10.8	29.9	68.6

Materials and Methods

For final rounds in which one will either win or lose, we use the standard winning probability expected from the Elo ratings,

$$P_{win}(\delta) = \frac{1}{1 + 10^{-\frac{\delta}{400}}}, \quad [1]$$

where $\delta = \text{Elo}^{\text{you}} - \text{Elo}^{\text{opponent}}$ is the difference of Elo ratings between the team of your interest and the opponent.

While probability of draw in football matches is considerable (25 ~ 30%) and that plays an important role for group league round results, Elo ratings gives only the probabilities of win and lose. Therefore, we here take the following draw probability function

$$P_{draw}(\delta) = \frac{0.3}{1 + \left(\frac{|\delta|}{300}\right)^2}, \quad [2]$$

to approximate the empirical probability of draw (2). The probabilities of win and lose a group league match is then assumed to be

$$P_{win}^{GL}(\delta) = P_{win}(\delta) - \frac{P_{draw}(\delta)}{2}, \quad [3]$$

so that the average change of Elo ratings after the match obeying this probability should be 0. As shown in Fig.1, these modified winning and losing probability functions take positive values at least in the range of $|\delta| < 600$, which covers all possible pairs in the group league round.

In the group league, the ranks among the teams with equal points are assigned randomly. This approximation is thought to help weaker teams because the ranks in such conditions in real competition is given according to the total goal difference, total goals, and so on. We assume no home-advantage for Qatar or other teams from Middle-East, that might be also unrealistic. For random number generator, we use Mersenne Twister (3).

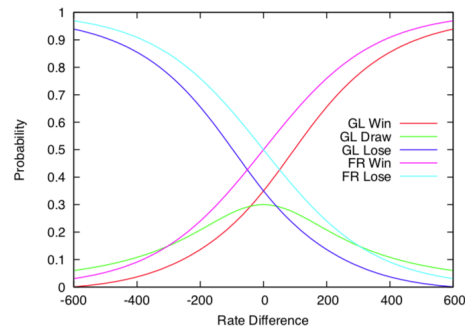


Fig. 1. Probability functions of win, draw, and lose for group league matches.

1. <https://www.eloratings>
2. <http://www.collective-behavior.com/publ/ELO.pdf>
3. M. Matsumoto & T. Nishimura, ACM Trans. on Modeling and Comp. Sim. 8. (1998) 3-30.

TS did everything.

Nothing to declare.

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Table 2. Estimated probabilities of reaching to each stage and the final positions in the group league (GL) round in the fixed Elo rating model, in %.

Country	Elo R.	Win the Cup	Final	R-4	R-8	R-16	GL: 1st	2nd	3rd	4th
QAT	1650	0.005 97	0.073	0.577	3.733	15.622	2.2	13.4	33.8	50.6
ECU	1840	0.616 59	2.942	10.296	32.457	67.350	17.9	49.4	22.7	9.9
SEN	1687	0.016 43	0.166	1.103	6.107	21.845	3.5	18.3	39.5	38.7
NLD	2040	8.796 68	16.924	34.842	72.035	95.183	76.4	18.8	4.0	0.8
ENG	1920	1.928 83	6.563	17.141	41.189	75.067	49.0	26.1	15.9	9.1
IRN	1817	0.240 38	1.196	4.742	17.405	46.298	19.7	26.6	27.4	26.3
USA	1798	0.153 24	0.825	3.555	14.149	40.484	16.3	24.2	28.3	31.2
WAL	1790	0.126 06	0.703	3.137	12.926	38.152	15.0	23.1	28.4	33.5
ARG	2141	22.319 70	32.701	53.932	73.140	97.365	85.6	11.8	2.4	0.2
SAU	1632	0.002 40	0.033	0.284	1.343	10.081	0.6	9.4	24.0	65.9
MEX	1813	0.264 79	1.482	5.689	14.003	46.937	7.0	39.9	36.5	16.6
POL	1809	0.241 82	1.379	5.377	13.395	45.616	6.7	38.9	37.2	17.2
FRA	2005	6.531 76	16.225	32.988	51.113	88.286	52.6	35.6	9.3	2.4
DEN	1971	4.024 94	11.081	25.181	42.353	83.705	41.3	42.4	12.6	3.7
TUN	1687	0.006 24	0.055	0.407	1.790	11.860	2.4	9.5	35.9	52.3
AUS	1719	0.015 62	0.119	0.748	2.862	16.149	3.6	12.5	42.2	41.7
ESP	2045	7.748 10	15.263	28.553	57.859	87.799	59.1	28.7	9.5	2.7
DEU	1960	2.638 09	7.429	16.114	38.656	73.261	31.4	41.9	19.2	7.5
JPN	1798	0.093 66	0.535	1.871	7.564	24.743	6.5	18.3	40.3	34.9
CRI	1737	0.019 41	0.149	0.654	3.372	14.197	3.1	11.1	31.0	54.8
BEL	2025	6.773 93	15.777	27.608	52.273	87.629	61.1	26.6	9.4	3.0
CAN	1770	0.041 98	0.240	1.094	5.743	23.464	6.0	17.4	35.7	40.8
MAR	1754	0.027 39	0.167	0.822	4.650	20.355	5.0	15.3	33.6	46.0
HRV	1922	1.372 01	4.336	10.772	29.883	68.551	27.9	40.7	21.3	10.2
BRA	2169	26.839 36	37.125	55.712	75.245	94.914	77.2	17.7	4.8	0.3
SRB	1893	0.737 22	2.907	7.291	18.820	44.818	9.4	35.5	44.6	10.6
CHE	1929	1.495 02	5.119	11.600	26.680	56.735	13.2	43.5	35.9	7.4
CMR	1613	0.000 48	0.008	0.055	0.455	3.533	0.2	3.3	14.6	81.8
PRT	2004	5.060 99	12.645	23.474	43.715	87.908	56.2	31.7	11.0	1.1
GHA	1541	0.000 05	0.001	0.013	0.154	2.968	0.4	2.6	15.7	81.3
URY	1936	1.794 61	5.469	12.819	28.909	77.484	34.5	42.9	19.7	2.9
KOR	1783	0.066 25	0.363	1.548	6.023	31.640	8.9	22.8	53.6	14.7

Table 3. Estimated probabilities in “dynamical-Elo-rating” model. Numbers are for reaching to each stage and the final positions in the group league (GL) round, in %.

Country	Elo R.	Win the Cup	Final	R-4	R-8	R-16	GL: 1st	2nd	3rd	4th
QAT	1650	0.016 37	0.122	0.761	4.290	16.778	2.7	14.1	33.3	49.9
ECU	1840	0.751 18	2.923	9.822	30.343	65.889	18.7	47.2	23.4	10.7
SEN	1687	0.038 05	0.249	1.360	6.713	23.043	4.1	19.0	38.6	38.4
NLD	2040	7.818 95	15.612	33.582	69.444	94.291	74.5	19.8	4.7	1.0
ENG	1920	2.198 76	6.941	17.410	40.549	73.455	47.0	26.4	16.6	9.9
IRN	1817	0.440 82	1.752	5.832	18.624	46.328	20.2	26.1	27.3	26.4
USA	1798	0.314 64	1.312	4.607	15.598	41.155	17.0	24.2	28.0	30.8
WAL	1790	0.272 55	1.159	4.159	14.439	39.062	15.8	23.3	28.1	32.9
ARG	2141	21.200 31	31.936	52.969	73.136	96.943	83.9	13.1	2.7	0.3
SAU	1632	0.005 33	0.052	0.398	1.700	10.826	0.8	10.0	24.9	64.3
MEX	1813	0.343 83	1.584	5.895	14.391	46.657	7.8	38.9	35.9	17.4
POL	1809	0.319 13	1.488	5.622	13.871	45.575	7.5	38.0	36.5	17.9
FRA	2005	6.520 49	15.786	31.324	49.417	86.715	51.8	34.9	10.4	2.9
DEN	1971	4.250 74	11.165	24.375	41.248	81.950	41.1	40.9	13.8	4.3
TUN	1687	0.020 46	0.127	0.701	2.473	13.429	2.9	10.6	35.1	51.5
AUS	1719	0.043 94	0.243	1.184	3.763	17.905	4.2	13.7	40.7	41.4
ESP	2045	7.453 24	14.512	27.665	56.195	86.276	57.6	28.7	10.5	3.2
DEU	1960	2.832 89	7.319	15.857	37.576	71.522	31.5	40.1	20.1	8.4
JPN	1798	0.181 09	0.790	2.449	8.842	26.455	7.3	19.2	38.8	34.8
CRI	1737	0.049 09	0.267	0.979	4.297	15.747	3.7	12.1	30.7	53.6
BEL	2025	6.611 81	15.192	26.851	51.077	86.169	59.1	27.1	10.3	3.5
CAN	1770	0.103 39	0.471	1.604	6.777	24.770	6.8	18.0	34.9	40.3
MAR	1754	0.073 72	0.353	1.258	5.625	21.731	5.7	16.0	33.1	45.2
HRV	1922	1.724 52	5.046	11.447	29.611	67.330	28.4	38.9	21.7	10.9
BRA	2169	26.482 28	36.051	53.823	74.699	94.372	75.8	18.6	5.2	0.4
SRB	1893	0.920 80	3.273	8.037	19.489	45.188	10.0	35.2	43.6	11.2
CHE	1929	1.733 73	5.423	12.136	26.766	56.456	13.9	42.5	35.6	7.9
CMR	1613	0.001 16	0.015	0.097	0.635	3.984	0.3	3.7	15.6	80.4
PRT	2004	5.175 67	12.426	22.753	42.551	86.909	55.1	31.8	11.7	1.4
GHA	1541	0.000 19	0.003	0.028	0.243	3.545	0.5	3.1	16.7	79.7
URY	1936	1.986 59	5.837	12.975	28.645	76.480	34.8	41.6	20.1	3.4
KOR	1783	0.114 29	0.571	2.041	6.971	33.066	9.6	23.5	51.4	15.5

Table 4. Appendix 1: Estimations of the earned points in the group round in % for the fixed Elo-rating model.

Country	Elo R.	9 pts. (3-0-0)	7 pts. (2-1-0)	6 pts. (2-0-1)	5 pts. (1-2-0)	4 pts. (1-1-1)	3 pts. (1-0-2)/(0-3-0)	2 pts. (0-2-1)	1 pt. (0-1-2)	0 pt. (0-0-3)
QAT	1650	0.2	0.9	4.7	1.4	12.9	23.0	8.5	26.4	22.1
ECU	1840	5.2	11.6	27.4	6.5	23.3	14.3	4.8	5.2	1.6
SEN	1687	0.4	1.7	7.3	2.2	17.1	25.7	8.9	22.3	14.4
NLD	2040	45.7	27.5	14.6	5.0	5.0	1.5	0.4	0.2	0.0
ENG	1920	15.2	22.2	17.7	10.8	17.2	8.6	4.2	3.3	0.9
IRN	1817	3.3	9.0	12.9	8.1	22.4	17.1	9.6	12.4	5.1
USA	1798	2.4	7.2	11.3	7.1	21.7	18.2	10.4	14.9	6.9
WAL	1790	2.1	6.5	10.6	6.6	21.3	18.5	10.6	16.0	7.8
ARG	2141	58.6	24.8	9.9	3.4	2.6	0.6	0.2	0.1	0.0
SAU	1632	0.0	0.3	2.3	0.6	7.9	18.2	6.6	28.3	35.7
MEX	1813	1.4	4.7	19.6	4.1	25.8	22.9	6.9	10.4	4.2
POL	1809	1.3	4.5	18.9	4.0	25.7	23.3	7.1	10.8	4.4
FRA	2005	24.1	27.1	22.9	7.8	11.4	4.4	1.3	0.8	0.2
DEN	1971	16.6	23.7	26.1	8.2	14.9	6.6	2.0	1.5	0.3
TUN	1687	0.2	0.9	3.8	1.4	11.5	23.7	7.8	27.0	23.8
AUS	1719	0.3	1.5	5.7	2.0	14.8	26.8	8.2	23.8	16.8
ESP	2045	26.9	27.3	19.9	8.0	11.0	4.3	1.4	0.9	0.2
DEU	1960	10.0	18.1	24.4	8.7	19.9	10.7	3.8	3.4	1.0
JPN	1798	0.8	2.9	8.5	3.4	18.8	25.2	9.0	19.8	11.6
CRI	1737	0.2	1.1	4.2	1.7	12.3	21.6	8.5	26.6	23.8
BEL	2025	27.5	27.2	19.1	8.2	11.0	4.4	1.5	1.0	0.2
CAN	1770	0.7	2.6	7.5	3.2	17.6	23.2	9.6	21.7	13.9
MAR	1754	0.5	2.0	6.3	2.7	15.9	22.5	9.5	23.7	17.0
HRV	1922	7.9	15.8	22.7	8.7	21.4	12.4	4.9	4.8	1.4
BRA	2169	49.2	27.0	13.7	4.5	4.3	1.2	0.2	0.1	0.0
SRB	1893	2.0	6.1	19.9	4.8	26.5	25.8	5.0	7.3	2.6
CHE	1929	3.5	9.0	25.5	5.8	26.1	19.7	4.1	4.8	1.5
CMR	1613	0.0	0.1	0.6	0.2	3.1	11.6	3.7	25.7	55.1
PRT	2004	27.6	28.0	21.4	7.5	10.3	3.8	0.9	0.5	0.1
GHA	1541	0.0	0.1	0.6	0.2	3.2	12.1	3.8	26.1	53.9
URY	1936	13.1	21.3	27.2	8.3	17.7	8.7	2.0	1.5	0.3
KOR	1783	1.5	4.8	13.8	4.4	23.8	31.5	5.5	10.3	4.3

Table 5. Appendix 2: Estimations of the earned points in the group round in % for the dynamical Elo-rating model. The Elo ratings in the parenthesis show the average of updated ratings after the championship.

Country	Elo R. (post)	9 pts. (3-0-0)	7 pts. (2-1-0)	6 pts. (2-0-1)	5 pts. (1-2-0)	4 pts. (1-1-1)	3 pts. (1-0-2)/(0-3-0)	2 pts. (0-2-1)	1 pt. (0-1-2)	0 pt. (0-0-3)
QAT	1650 (1647)	0.4	1.4	5.2	1.7	12.8	20.7	8.2	25.7	24.0
ECU	1840 (1832)	6.4	12.4	26.1	6.1	21.8	13.5	5.1	6.2	2.4
SEN	1687 (1683)	0.6	2.3	7.7	2.6	16.6	23.4	8.4	22.1	16.3
NLD	2040 (2044)	46.9	26.4	13.3	5.1	5.4	1.8	0.6	0.4	0.1
ENG	1920 (1916)	17.8	21.7	15.2	10.2	15.9	8.5	4.8	4.4	1.5
IRN	1817 (1811)	4.7	10.2	11.9	8.0	20.1	15.4	9.4	13.4	6.9
USA	1798 (1792)	3.5	8.5	10.5	7.3	19.5	16.3	9.9	15.5	9.0
WAL	1790 (1785)	3.1	7.8	9.9	7.0	19.2	16.5	10.1	16.5	9.9
ARG	2141 (2167)	59.2	24.0	9.2	3.5	2.9	0.8	0.2	0.1	0.0
SAU	1632 (1631)	0.1	0.4	2.8	0.8	8.4	16.7	6.6	27.0	37.2
MEX	1813 (1808)	2.0	5.5	19.5	4.1	24.3	21.1	6.9	11.4	5.2
POL	1809 (1804)	1.9	5.4	18.9	4.0	24.1	21.5	7.0	11.7	5.5
FRA	2005 (2011)	25.8	26.4	20.7	7.5	11.4	4.9	1.7	1.3	0.3
DEN	1971 (1973)	18.3	23.4	23.8	7.8	14.7	6.9	2.4	2.1	0.6
TUN	1687 (1686)	0.3	1.3	4.2	1.7	11.5	21.5	7.6	26.3	25.5
AUS	1719 (1717)	0.6	2.1	6.0	2.4	14.6	24.5	7.8	23.5	18.5
ESP	2045 (2049)	28.8	26.4	17.6	7.9	11.0	4.7	1.8	1.4	0.4
DEU	1960 (1957)	11.7	18.3	22.2	8.2	18.8	10.5	4.3	4.4	1.5
JPN	1798 (1794)	1.3	3.8	8.5	3.9	17.9	22.9	8.5	19.8	13.5
CRI	1737 (1735)	0.5	1.7	4.6	2.1	12.2	19.3	8.2	25.8	25.7
BEL	2025 (2029)	29.6	26.0	17.0	7.9	10.8	4.8	1.9	1.5	0.4
CAN	1770 (1766)	1.1	3.4	7.7	3.5	16.5	21.0	9.1	21.7	16.0
MAR	1754 (1751)	0.9	2.8	6.5	3.0	15.1	20.2	9.1	23.3	19.2
HRV	1922 (1917)	9.6	16.6	20.7	8.3	19.6	11.9	5.1	5.8	2.2
BRA	2169 (2203)	50.3	26.0	12.4	4.6	4.7	1.5	0.4	0.2	0.0
SRB	1893 (1891)	2.7	6.8	19.2	4.8	25.1	25.0	4.9	8.0	3.4
CHE	1929 (1928)	4.4	9.7	24.3	5.6	24.8	19.4	4.2	5.5	2.0
CMR	1613 (1613)	0.0	0.1	0.8	0.3	3.5	10.7	3.8	24.8	56.0
PRT	2004 (2006)	29.3	27.0	19.6	7.3	10.6	4.4	1.1	0.7	0.1
GHA	1541 (1541)	0.0	0.1	0.8	0.3	3.6	11.3	4.0	25.2	54.6
URY	1936 (1933)	14.6	21.5	24.8	8.0	17.1	9.2	2.2	2.0	0.6
KOR	1783 (1780)	2.1	5.8	13.1	4.7	22.5	30.3	5.3	11.0	5.3